Technical Note

1965-22

R. Price

Statistical Synthesis
of a pP-Wave Enhancer

ESRL

14 June 1965

AD0617951

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

STATISTICAL SYNTHESIS OF A pP-WAVE ENHANCER

R. PRICE

Group 64

TECHNICAL NOTE 1965-22

14 JUNE 1965

LEXINGTON   MASSACHUSETTS

Statistical Synthesis of a pP-Wave Enhancer

R. Price

Lincoln Laboratory, Massachusetts Institute of Technology
Lexington, Massachusetts

ABSTRACT

This note provides a communication-theoretic rationale for a successful nonlinear processing scheme due to Shimshoni and Smith, by showing its resemblance to minimum-rms-error estimation. The latter filtering, although "optimum" under certain assumptions, is rather more complicated in its implementation.

As in all such filtering studies, the mean-square error is made up of bias, or signal distortion, and variance (which in linear filtering is attributable only to the noise). It would be interesting to see whether the filtering developed here could be modified to allow a controlled tradeoff between these two sources of error.

Shimshoni and Smith[1] have described a procedure that takes advantage of the linear polarization in a P-wave arrival to enhance its visibility against noise that is assumed to be unpolarized. Their technique is to multiply the output of the vertical seismometer by a running history of the short-term, zero-lag crosscorrelation taken between the vertical and horizontal seismometer outputs, with the horizontal seismometer aligned to the radial component of P-wave particle motion.

The purpose of his note is to demonstrate that while the Shimshoni-Smith procedure may at first seem quite ad hoc, it is in fact "cousin" to a nonlinear processing scheme which, under assumptions to be given, is optimal in the sense that its output is (proportional to) the minimum-variance estimate of the P-wave signature. Although it may not be worthwhile to implement this optimum scheme, its similarity to the Shimshoni-Smith procedure at least can serve to explain the latter's marked success and support its philosophy.

We model the problem as follows. The outputs of the vertical and horizontal seismometers are assumed to contain independent gaussian noises (i.e., the noise in the particle motion is unpolarized) that are white and of equal intensity (these restrictions are not essential, but are made merely for analytical convenience in this cursory study). The P-wave is taken to be the result of translating a sample of non-stationary gaussian noise, which initially vanishes outside an interval $(0, \Delta)$ and has a known correlation function $\varphi_P(t, \tau)$ inside, by an unknown amount t_S, so that the (unknown-waveform) P-wave signature occupies the interval $(t_S, t_S + \Delta)$. Its starting time t_S is assumed to have a known a priori probability density over some finite interval $(t_0, t_0 + T)$, which vanishes outside. The vertical and horizontal seismometers are constructed to respond identically to the arriving P-wave except for sine and cosine gain factors that depend on the arrival angle. To form a minimum-variance estimate of the P-wave signature, one is forced, as in Wiener linear filtering, to assume some a priori statistics for unknown quantities as above; if the arrival angle is unknown as well, we must likewise assign it an a priori distribution.

At least in the important case of discerning the pP arrival, it appears safe to assume that the arrival angle is known, this information being provided by the generally stronger main-P arrival. We shall depart from the Shimshoni-Smith context by assuming the arrival angle to be known. If it is indeed unknown, one would obtain the minimum-variance estimate by applying the following analysis to each of a continuum of hypothesized arrival angles, and then averaging the resulting infinitely-many minimum-variance estimates (each conditioned on a different hypothesized angle) over the a priori distribution of angles.

At the outset, we know that the minimum-variance pP-wave signature estimator must be nonlinear, for the signature has non-gaussian statistics as a result of the averaging of its conditionally gaussian (on t_S) statistics over the random starting time t_S. Hence there should be no prejudice a priori against procedures like those of Shimshoni-Smith. However, we want to see where we are led through a synthesis of the nonlinear processing from basic statistical principles (admitting that this requires questionable assumptions of a priori statistics), avoiding ad hoc notions even though the outcome of this abstract synthesis may have a straightforward heuristic interpretation.

To begin, we are given the respective outputs $w_V(t)$ and $w_H(t)$ of the vertical and horizontal seismometers, observed over all time (this assumption is not essential). Knowing the pP arrival angle, we may form a new pair of waveforms $w(t)$ and $\tilde{w}(t)$ through a (time-independent) rotational transformation of $w_V(t)$ and $w_H(t)$, in such a way that $\tilde{w}(t)$ contains only noise (independent of that in $w(t)$ by virtue of the assumed equal noise intensities for $w_V(t)$ and $w_H(t)$ and the fact that the transformation is a rotation) and no pP-wave energy. (In effect, we construct a new pair of orthogonally aligned seismometers, the one whose output is $\tilde{w}(t)$ responding only at right-angles to the direction of signal particle motion.) Nothing is lost in performing such a rotation, for it is a reversible operation; therefore, the minimum P-signature error variance obtainable by observing $w(t)$ and $\tilde{w}(t)$ is identical to that obtainable by observing the original outputs $w_V(t)$ and $w_H(t)$.

It is a universal rule, irrespective of whether or not the statistics that are involved are gaussian, that the minimum-variance estimator (which generally must suffer some bias or signal distortion, as in Wiener estimation) always computes the conditional mean of the signal whose estimate is sought, where the conditioning is on whatever observations are available. Thus our optimum estimator is

$$\hat{x}_{W,\tilde{W}}(t) = \overline{x(t)}\Big|_{W,\tilde{W}} = \int_{-\infty}^{\infty} x_t \, p[x_t/W,\tilde{W}] \, dx_t \tag{1}$$

where x(t) is the pP-wave signature whose minimum-variance estimate is sought from the observations of w(t) and $\tilde{w}$(t), and this latter pair of histories is for conciseness denoted by W and $\tilde{W}$, respectively. (In averaging, the notation x_t rather than x(t) is used to indicate that in this instance we are concerned with only a single instant of time.)

We now have recourse to the result

$$p[x_t/W,\tilde{W}] = \frac{p[W,\tilde{W},x_t]}{p[W,\tilde{W}]} = \frac{p[\tilde{W}] \, p[W,x_t]}{p[\tilde{W}] \, p[W]} = p[x_t/W] \tag{2}$$

Philosophically, the fact that $\tilde{w}$(t) contains only noise and that this noise is independent of that in w(t) causes it to play no role in the estimation of the P-wave signature, as has just been demonstrated mathematically.

Using the identity:

$$p[x_t/W] = \int_{t_o}^{t_o+T} p[x_t/W,t_s] \, p[t_s/W] \, dt_s \tag{3}$$

and Bayes's rule:

$$p(t_s/W) = \frac{p(W/t_s) \, p(t_s)}{p(W)} \tag{4}$$

our estimate becomes

$$\hat{x}_{W,\tilde{w}}(t) = \hat{x}_{W}(t) = [p(W)]^{-1} \int_{t_0}^{t_0+T} \left\{ \int_{-\infty}^{\infty} x_t \, p[x_t/W,t_s] \, dx_t \right\} p(W/t_s) \, p(t_s) \, dt_s \qquad (5)$$

The braced inner integral in (5), which may conveniently be denoted by $\hat{x}_{W,t_s}(t)$, is identical to the linear Wiener estimate of $x(t)$, conditioned on both the observation of $w(t)$ and the hypothesis that the starting time for $x(t)$ is t_s, for it can be shown that $p[x_t/W,t_s]$ is gaussian in x_t and thence that

$$\hat{x}_{W,t_s}(t) = \int_{-\infty}^{\infty} w(\tau) \, h(\tau-t_s, t-t_s) \, d\tau \qquad (6)$$

where the (symmetric, nonrealizable) linear operator $h(\tau,t)$ solves the integral equation

$$\int_{0}^{\Delta} [\varphi_p(t,\tau) + (N_0/2) \, \delta(t-\tau)] \, h(\tau,\sigma) \, d\tau = \varphi_p(\tau,\sigma) \; ; \; 0 \le t, \sigma \le \Delta \qquad (7)$$

and vanishes for either τ or t outside $(0,\Delta)$. Here $\varphi_p(t,\tau)$ is the correlation function of the gaussian P-wave signature for zero translation ($t_s = 0$), $\delta(t)$ is the Dirac delta-function or unit impulse, and N_0 is the (single-sided, physical) spectral density of the white noise in the observation $w(t)$.

It is now clear from (5) that, taking into account the dependence of $\hat{x}_W(t)$ on $p(W/t_s)$ as well as (linearly) on $w(t)$ through $\hat{x}_{W,t_s}(t)$, the minimum-variance estimator will be nonlinear in $w(t)$. (Henceforth, we shall ignore the outside factor $[p(W)]^{-1}$ in (5), as for any given observation it represents a fixed gain. Thus our estimate will now be proportional to the minimum-variance estimate, rather than strictly equaling it--additional computation could supply $p(W)$ if it were needed, but ordinary AGC along with compensation by the human eye should suffice unless the magnitude of pP must be estimated.)

It remains to determine $p(W/t_s)$, the a priori probability of obtaining the given observation under the hypothesis that the (conditionally) gaussian P-wave signature commences at t_s. Applying the results for the detection of a gaussian signal in white gaussian noise (e.g., Price, IRE Trans. on Info. Theory, December 1956), we have

$$p(W/t_s) \sim \exp \left\{ \frac{1}{N_o} \int_{-\infty}^{\infty} w(t) \left[\int_{-\infty}^{\infty} w(\tau) h(\tau - t_s, t - t_s) d\tau \right] dt \right\} \tag{8}$$

where again we are not concerned with the (here $w(t)$-independent) constant of proportionality. Assembling (5)-(8), we arrive at the block diagram (page 7) for the processor whose output is proportional to the minimum-variance estimate of the P-wave, and which exhibits linear and quadratic operations, and a "bootstrap" connection between them not unlike that of Shimshoni-Smith.

With regard to the quadratic operation, Shimshoni-Smith multiply the original seismometer outputs $w_V(t)$ and $w_H(t)$ and integrate the product for a short period. Recalling that the input to the processor shown in the figure results from a rotation and is thus of the form $w(t) = aw_V(t) + bw_H(t)$, we note that such a product (with some filtering) is among the components appearing at the output of the uppermost multiplier, and that for any given t_s the indicated infinite-time integration is in fact only finite by virtue of the Δ-limited filter memories.

REFERENCE

1. M. Shimshoni and S. W. Smith, Seismic signal enhancement with 3-component detectors, Geophysics, October 1964, pp. 664-671. This procedure was described independently by C. H. Mims and R. L. Sax in Rectilinear motion detection, paper presented at the Seismological Society of America Meeting, St. Louis, Missouri, April 1965.
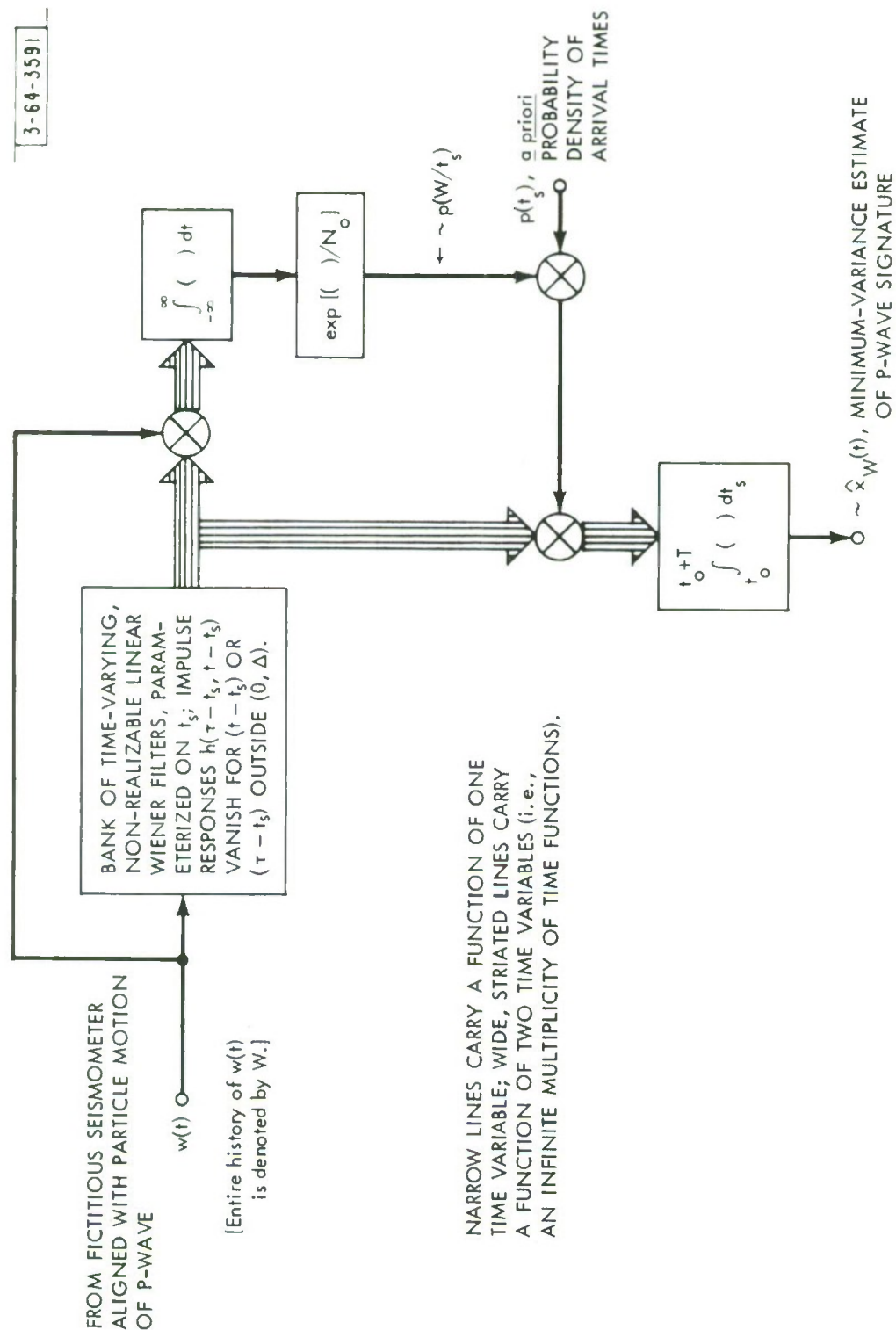
3-64-3591

FROM FICTITIOUS SEISMOMETER
ALIGNED WITH PARTICLE MOTION
OF P-WAVE

w(t)

[Entire history of w(t)
is denoted by W.]

BANK OF TIME-VARYING,
NON-REALIZABLE LINEAR
WIENER FILTERS, PARAM-
ETERIZED ON t_s; IMPULSE
RESPONSES $h(\tau - t_s, t - t_s)$
VANISH FOR $(t - t_s)$ OR
$(\tau - t_s)$ OUTSIDE $(0, \Delta)$.

$$\int_{-\infty}^{\infty} (\ ) \, dt$$

$$\exp[(\ )/N_o]$$

$\sim p(W/t_s)$

$p(t_s)$, a priori
PROBABILITY
DENSITY OF
ARRIVAL TIMES

$$\int_{t_o}^{t_o+T} (\ ) \, dt_s$$

$\sim \hat{x}_W(t)$, MINIMUM-VARIANCE ESTIMATE
OF P-WAVE SIGNATURE

NARROW LINES CARRY A FUNCTION OF ONE
TIME VARIABLE; WIDE, STRIATED LINES CARRY
A FUNCTION OF TWO TIME VARIABLES (i.e.,
AN INFINITE MULTIPLICITY OF TIME FUNCTIONS).

Figure 1.   Optimum nonlinear processor for obtaining pP-wave signature.

6

DOCUMENT CONTROL DATA – R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*	2a. REPORT SECURITY CLASSIFICATION
Lincoln Laboratory, M.I.T.	Unclassified
	2b. GROUP
	None

3. REPORT TITLE

Statistical Synthesis of a pP-Wave Enhancer

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Technical Note

5. AUTHOR(S) *(Last name, first name, initial)*

Price, Robert

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
14 June 1965	9	1

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
AF 19(628)-500	
b. PROJECT NO.	Technical Note 1965-22
ARPA Order 512	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	ESD-TDR-65-235

10. AVAILABILITY/LIMITATION NOTICES

None

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None	Advanced Research Projects Agency, Department of Defense

13. ABSTRACT

This note provides a communication-theoretic rationale for a successful nonlinear processing scheme due to Shimshoni and Smith, by showing its resemblance to minimum-rms-error estimation. The latter filtering, although "optimum" under certain assumptions, is rather more complicated in its implementation. As in all such filtering studies, the mean-square error is made up of bias, or signal distortion, and variance (which in linear filtering is attributable only to the noise). It would be interesting to see whether the filtering developed here could be modified to allow a controlled tradeoff between these two sources of error.

14. KEY WORDS

statistical analysis   nonlinear systems   filters